PRACTICE FINAL EXAM Solutions (F 06)

Domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$ because x^2-1 cannot equal to zero. Denominator $\neq 0$.

$$f'(x)=\frac{(x^2-1)(0)-(1)(2x)}{(x^2-1)^2}$$

$$=\frac{0-2x}{(x^2-1)^2}=\frac{-2x}{(x^2-1)^2}$$

Find possible critical numbers and determine whether or not they are actual critical numbers.

Where does $f'(x)=0$? (ZERO)

$$f'(x)=\frac{-2x}{(x^2-1)^2}=0$$

$-2x=0$ (Numerator $=0$)

$x=0$

0 is a C# b/c it is defined in the domain.

Verification:

$f(0)=\frac{1}{(0)^2-1}=\frac{1}{-1}=-1$ (Solution) Passed

Where does $f'(x)$ not exist? (DNE)

$(x^2-1)^2=0$ (Denominator $=0$)

$x^2-1=0$ $\quad (x-1)(x+1)=0$

$x=1$; $x=-1$

Both of these values are ommitted from the domain of the original function, $f(x)$.

Verification:

$f(-1)=\frac{1}{(-1)^2-1}=\frac{1}{1-1}=\frac{1}{0}$ Failed — zero in the denominator

$f(1)=\frac{1}{(1)^2-1}=\frac{1}{(1)-1}=\frac{1}{0}$ Failed

Neither of these can be critical #'s.

a) The only critical number is 0, but you still use -1 and 1 to set up the intervals.

$(-\infty,-1)$		$(-1,0)$		$(0,1)$		$(1,\infty)$
$+\nearrow$	-1 (V.A.)	$+\nearrow$	0 (C#)	$-\searrow$	1 (V.A.)	$-\searrow$

$f(x)$ is INC; $f'(x)$ is $+$

Test for the interval $(-\infty,-1)$ using -2 as the test #:

$$f'(-2)=\frac{-2(-2)}{((-2)^2-1)^2}=\frac{4}{3}\ \oplus$$

Test for the interval $(-1,0)$ using $-\frac{1}{2}$ as the test #:

$$f'(-\tfrac{1}{2})=\frac{-2(-\frac{1}{2})}{((-\frac{1}{2})^2-1)^2}=\frac{\oplus}{\oplus}=+$$

Test for the interval $(0,1)$ using $\frac{1}{2}$ as the test #:

$$f'(\tfrac{1}{2})=\frac{-2(\frac{1}{2})}{((\frac{1}{2})^2-1)^2}=\frac{\ominus}{\oplus}=-$$

Test for the interval $(1,\infty)$ using 2 as the test #:

$$f'(2)=\frac{-2(2)}{((2)^2-1)^2}=\frac{\ominus}{\oplus}=-$$

b) $(-\infty,-1)$ and $(-1,0)$. The intervals are separated b/c $-1=x$ is a V.A.

c) $(0,1)$ and $(1,\infty)$. The intervals are separated b/c $1=x$ is a V.A.

d) Relative maximum:

$f(0)=\frac{1}{(0)^2-1}=\frac{1}{-1}=-1$

i. Relative maximum of -1 at $x=0$.

ii. Relative minimum of DNE at $x=$ ___ .

To find points of inflection:

$$f''(x)=\frac{(x^2-1)^2(-2)-[(-2x)(2)(x^2-1)^1(2x)]}{(x^2-1)^4}$$

$$=\frac{(x^2-1)(-2)-[(-2x)(2)(2x)]}{(x^2-1)^3}$$

$$=\frac{-2x^2+2+8x^2}{(x^2-1)^3}=\frac{6x^2+2}{(x^2-1)^3}$$

Determine where $f''(x)=0$.

$$f''(x)=\frac{6x^2+2}{(x^2-1)^3}=0$$

$6x^2+2=0$

$2(3x^2+1)=0$ $\quad 3x^2+1=0$

$3x^2=-1$ $\quad x^2=-\frac{1}{3}$ (No possible solution.)

* You may verify with your graphing calculator.

Determine where $f''(x)$ DNE.

$(x^2-1)^3=0$ $\quad x^2-1=0$

$x^2=1$ $\quad \sqrt{x^2}=\sqrt{1}$ $\quad = x=\pm 1$

There will not be any points of inflection because there aren't any defined values to even test. (Reasoning)

Here is the table any way to verify:

$(-\infty,-1)$		$(-1,1)$		$(1,\infty)$
$+$ C. up	-1 V.A.	$-$ C. down	1 V.A.	$+$

Test for the interval $(-\infty,-1)$ using -2:

$$f''(-2)=\frac{6(-2)^2+2}{((-2)^2-1)^3}=\frac{\oplus}{\oplus}=+$$

Test for the interval $(-1,1)$ using 0:

$$f''(0)=\frac{6(0)^2+2}{((0)^2-1)^3}=\frac{\oplus}{\ominus}=-$$

Test for the interval $(1,\infty)$ using 2:

$$f''(2)=\frac{6(2)^2+2}{((2)^2-1)^3}=\frac{\oplus}{\oplus}=+$$

Note that the function does change concavity at -1 and 1, but they are both vertical asymptotes.

e) There are no points of inflection.

2. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

3. a) $\lim_{x\to -3} \dfrac{(x+3)(x-3)}{(x+3)(x-2)}$

$\lim_{x\to -3} \dfrac{x-3}{x-2} = \dfrac{-3-3}{-3-2} = \dfrac{-6}{-5} = \dfrac{6}{5}$

b) $\lim_{x\to 7} \dfrac{2x^3-9}{x+10} = \dfrac{2(7)^3-9}{7+10}$

c) $\lim_{x\to\infty} \dfrac{3x^3+2x-1}{2x^4-3x^3-2} = 0$

$\overset{3}{DN} < \overset{4}{DD}$

d) $\lim_{x\to\infty} \dfrac{x^4-16}{5-x^2} = DNE$

$\overset{4}{DN} > \overset{2}{DD}$

4. a. 2   a. 1
b. 0   b. 1
c. DNE   c. 1
d. 0   d. 3

Discontinuous at
0, 4, 5, 7

5. $f(x) = [\ln(11x^8-2.5x^2-x)](e^{x^4+4x+1})$

$F = \ln(11x^8-2.5x^2-x)$ $S = e^{x^4+9x+1}$

$F' = \dfrac{88x^7-5x-1}{11x^8-2.5x^2-x}$ $S' = (4x^3+9)e^{x^4+9x+1}$

a) $f'(x) = FS' + SF'$

$= [(\ln(11x^8-2.5x^2-x))(4x^3+9)e^{x^4+9x+1}] + (e^{x^4+9x+1})\left(\dfrac{88x^7-5x-1}{11x^8-2.5x^2-x}\right)$

b) $f'(x) = 5(19x^3-18x^2-17x+99)^4(57x^2-36x-17)$

$B^2 = (\sqrt{x})^2 = (x^{1/2})^2 = x$

c) $T = 6x^{1/3}-51$ $B = \sqrt{x} = x^{1/2}$

$T' = 2x^{-2/3}$ $B' = \frac{1}{2}x^{-1/2}$

Simplifications are optional.

$f'(x) = \dfrac{(x^{1/2})(2x^{-2/3}) - [(6x^{1/3}-51)(\frac{1}{2}x^{-1/2})]}{x}$

d) $y = \sqrt{15x^2-26x+\pi} = (15x^2-26x+\pi)^{1/2}$

$y' = \dfrac{dy}{dx} = \frac{1}{2}(15x^2-26x+\pi)^{-1/2}(30x-26)$

6.a) $F(x) = \dfrac{11x^3}{3} - e^x + 8\ln x + C$

6. $\int e^{9x} + \sqrt{x} - \dfrac{3}{x^4}\,dx$

$= \int e^{9x} + x^{1/2} - 3x^{-4}\,dx$

$= \dfrac{e^{9x}}{9} + \dfrac{x^{1/2+1}}{1/2+1} - 3\left[\dfrac{x^{-4+1}}{-4+1}\right] + C$

$= \dfrac{e^{9x}}{9} + \dfrac{x^{3/2}}{3/2} - 3\left[\dfrac{x^{-3}}{-3}\right] + C$

$= \dfrac{e^{9x}}{9} + \dfrac{2}{3}x^{3/2} + x^{-3} + C$

7. a. $\int (x+3)(x^2+6x)^{-4}\,dx$

$u = x^2+6x$

$u' = \dfrac{du}{dx} = 2x+6$

$du = (2x+6)\,dx$ ⊛

$dx = \dfrac{du}{(2x+6)}$ ⊛

$\int (x+3)(u)^{-4}\left(\dfrac{du}{2x+6}\right)$

$\int (x+3)(u^{-4})\left(\dfrac{du}{2(x+3)}\right)$

$\int u^{-4}\dfrac{du}{2}$

$\int u^{-4}(\frac{1}{2})\,du$

$\frac{1}{2}\int u^{-4}\,du$

$\frac{1}{2}\left(\dfrac{u^{-3}}{-3}\right) + C$

$-\frac{1}{6}u^{-3} + C$

$-\frac{1}{6}(x^2+6x) + C$
(Solution)

7b. $\int (6x-9)e^{3x^2-9x+8}\,dx$

$u = 3x^2-9x+8$

$u' = \dfrac{du}{dx} = 6x-9$

$du = (6x-9)\,dx$ ⊛

$dx = \dfrac{du}{(6x-9)}$ ⊛

$\int (6x-9)e^{(u)}\left(\dfrac{du}{6x-9}\right)$

$\int e^u\,du = e^u + C$

$e^{(3x^2-9x+8)} + C$
(solution)

8. a. $\int_2^7 (x^3-15x^2)\,dx$

$\dfrac{x^4}{4} - \dfrac{15x^3}{3} = \dfrac{3x^4}{12} - \dfrac{60x^3}{12}$

$\dfrac{3x^4-60x^3}{12} = \left.\dfrac{x^4-20x^3}{4}\right|_2^7$

$F(x) = \frac{1}{4}(x^4-20x^3)$

$F(7) = \frac{1}{4}(7^4-20(7)^3) = \dfrac{-4459}{4}$

$F(2) = \frac{1}{4}(2^4-20(2)^3) = -36$

$F(7) - F(2) = \dfrac{-4315}{4} = -1078.75$

* Your solution should be the same if you correctly used

$\left.\dfrac{x^4}{4} - \dfrac{15x^3}{3}\right|_2^7$

or

$\left.\dfrac{x^4}{4} - 5x^3\right|_2^7$

9. $\int_{-1}^{1} (4x+10)(x^2+5x+3)^2\,dx$

$u = x^2+5x+3$

When $x=1$, $u = (1)^2+5(1)+3 = 9 =$ "b"

When $x=-1$, $u = (-1)^2+5(-1)+3 = -1 =$ "a"

$\dfrac{du}{dx} = u' = 2x+5$

$du = (2x+5)\,dx$ ⊛

$dx = \dfrac{du}{(2x+5)}$

$\int_{-1}^{9} (4x+10)(u)^2\left(\dfrac{du}{(2x+5)}\right)$

$\int_{-1}^{9} 2(2x+5)(u)^2\left(\dfrac{du}{(2x+5)}\right)$

$\int_{-1}^{9} 2(u)^2\,du = 2\int_{-1}^{9} u^2\,du$

$\left.2\left(\dfrac{u^3}{3}\right)\right|_{-1}^{9}$ $F(u) = \dfrac{2u^3}{3}$

$F(9) = \dfrac{2(9)^3}{3} = 486$

$F(-1) = \dfrac{2(-1)^3}{3} = \dfrac{-2}{3}$

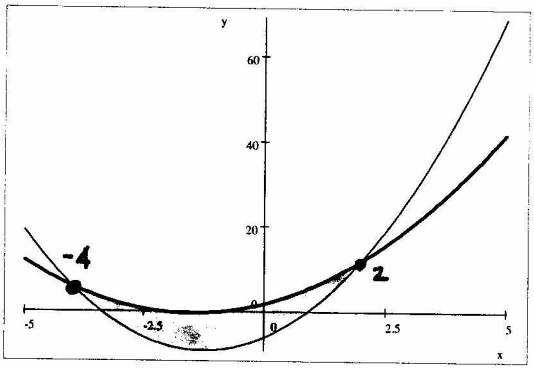
$F(9) - F(-1) = 486 - (\frac{-2}{3})$

$= \dfrac{1460}{3}$

(9)

$y = 2x^2 + 5x - 6$ (This has the thin line.)

$\mathbf{g = x^2+3x+2}$ (This has the thick line.)



Refer to examples where a and b are <u>NOT</u> given.

$2x^2 + 5x - 6 = \mathbf{x^2+3x+2}$, Solution is: $-4, 2$

The points of intersection are -4 and 2.

(**TOP**)-(Bottom)

$(\mathbf{x^2+3x+2}) - (2x^2 + 5x - 6) = -x^2 - 2x + 8$

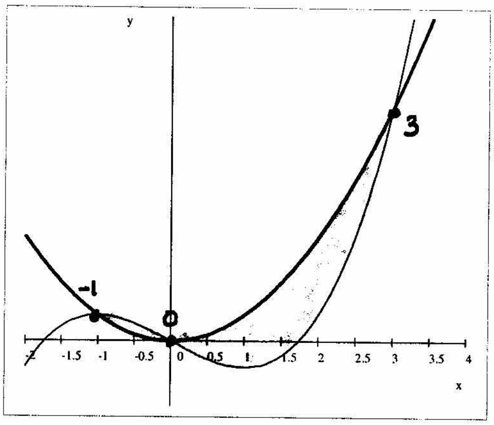
$\int_{-4}^{2}(-x^2 - 2x + 8)dx = 36$ ← This is the solution but you must find the antiderivative, F(x), First then evaluate F(2)-F(-4).

$-\frac{x^3}{3} - x^2 + 8x \Big|_{-4}^{2}$

(10)

$y = x^3 - 3x$

$y = 2x^2$



Refer to examples where a and b are <u>NOT</u> given.

$x^3 - 3x = 2x^2$, Solution is: $3, 0, -1$

The points of intersection are -1,0 and 3.

AREA 1: (**TOP**)-(Bottom) over the interval [-1, 0]

$(x^3 - 3x) - (2x^2) = x^3 - 2x^2 - 3x$

$\int_{-1}^{0}(x^3 - 2x^2 - 3x)dx \qquad \frac{x^4}{4} - \frac{2x^3}{3} - \frac{3x^2}{2} \Big|_{-1}^{0}$

Area 1 = $\frac{7}{12} = 0.58333$

AREA 2: (**TOP**)-(Bottom) over the interval [0, 3]

$(2x^2) - (x^3 - 3x) = 3x + 2x^2 - x^3$

$\int_{0}^{3}(3x + 2x^2 - x^3)dx \qquad \frac{3x^2}{2} - \frac{2x^3}{3} - \frac{x^4}{4} \Big|_{0}^{3}$

Area 2 = $\frac{45}{4} = 11.25$

Total AREA =(AREA 1) + (AREA 2)

$\frac{7}{12} + \frac{45}{4} = \frac{71}{6} = 11.833$

(11)
Profit = Revenue - Cost
$P(x) = (28x) - (x^3 - 6x^2 + 13x + 15) = -x^3 + 6x^2 + 15x - 15$
First Derivative:
$\frac{d}{dx}(-x^3 + 6x^2 + 15x - 15) = -3x^2 + 12x + 15$
Find the critical numbers. (The domain of a polynomial function is all real numbers, so at any value where the first derivative equals to zero that value will be a critical number.)
$-3x^2 + 12x + 15 = 0$, Solution is: $-1, 5$
$-3(x^2 - 4x - 5) = 0; -3(x-5)(x-1) = 0$
Note that the interval specified is concerned only with values over the interval [0, 8].
Therefore, the critical number -1 must be excluded because it is not within the given interval.
Now, for example, if the interval was [-2,9] both critical numbers would be used.
Plug the critical number 5 and the endpoints into the equation $P(x) = -x^3 + 6x^2 + 15x - 15$.

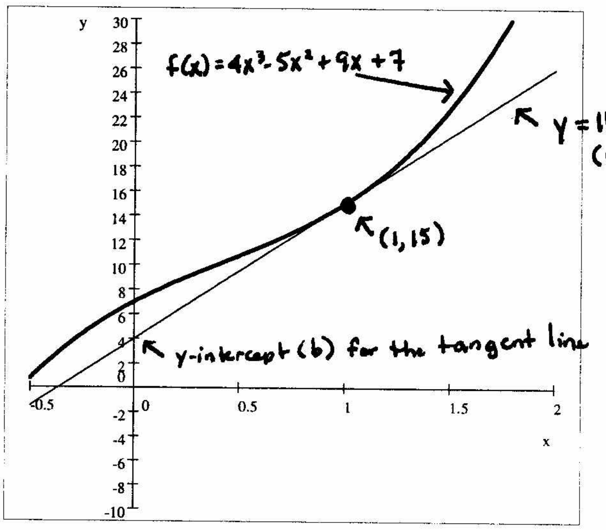
x	P(x)
5	85
0	−15
8	−23

Assuming profit is in units of ten dollars,
The profit is $ 850 when 5 units are produced.

(12) Read the text covering section 6.1.

(13)
We know that x = 1, so what is y equal to?
$y = f(1) = [4(1)^3 - 5(1)^2 + 9(1) + 7] = 15$
So we know know that the tagent line will touch the function only at the point (1, 15).
Now we need to determine the slope. First find the first derivative so that we can determine the slope to the curve when x = 1.
The first derivative: $\frac{d}{dx}[4(x)^3 - 5(x)^2 + 9(x) + 7] = [12x^2 - 10x + 9]$
The first derivative evaluated when x = 1: $f'(1) = [12(1)^2 - 10(1) + 9] = 11$.
It was evaluated at 1, because we want to know the slope of the tangent line when x specifically is 1.
Also note that at the point the function must be in an increasing direction, because the slope is positive.

Now use one of the equations to find the equation of a line using the slope=11 and point (1,15).
Your answer should be $y_{Tan} = 11x + 4$.



(14) Refer to lecture notes, textbook (section 3.4 example 5 pg. 186), ect. Your final answer should be $f'(x) = 2x - 3$.

(15)

$f(x) = -3x^4 + 8x^3 + 5$

a. The domain of the function is all real numbers b/c it is a polynomial.

b. There are not vertical asymptotes b/c it is defined for all values of x.

c. There is not a horizontal asympotes, b/c the limit as x approahes infinity does not exist.

d. The first derivative: $\frac{d}{dx}(-3x^4 + 8x^3 + 5) = 24x^2 - 12x^3 = (-12)(x^2)(x-2)$

e. The critical numbers are when the polynomial equals to zero, b/c where the first derivative does not exist does not apply b/c the domain of the polynomial is all real number.

$-12x^3 + 24x^2 = (-12)(x^2)(x-2) = 0$, Solution is: $0, 2$

The critical numbers are 0 and 2.

Intervals: $(-\infty, 0)(0, 2)(2, \infty)$

Test for the interval $(-\infty, 0)$ using x= -3:$-12(-3)^3 + 24(-3)^2 = 540 \quad (+)$

The slope of any tangent line in this interval is positive therefore the function is increasing.

Test for the interval $(0, 2)$ using x = 1:$-12(1)^3 + 24(1)^2 = 12 \quad (+)$

The slope of any tangent line in this interval is positive therefore the function is increasing.

Test for the interval $(2, \infty)$ using x = 15: $-12(15)^3 + 24(15)^2 = -35\,100 \quad (-)$

The slope of any tangent line in this interval is negative therefore the function is decreasing.

f. $(-\infty, 2)$ Remember the function is continuous at x = 0. Compare this to problem #1 and other previous examples.

g. $(2, \infty)$

h. The second derivative: $f''(x) = \frac{d}{dx}(-12x^3 + 24x^2) = 48x - 36x^2 = (-12)(x)(3x-4)$

Find where it equals to zero: $(-12)(x)(3x-4) = 0$, Solution is: $0, \frac{4}{3}$ These values at this step does not guarantee x -values for point(s) of inflection.

Intervals: $(-\infty, 0)(0, \frac{4}{3})(\frac{4}{3}, \infty)$

Test for the interval $(-\infty, 0)$ using x= -3:$(-12)(-3)(3(-3)-4) = -468 \quad (-)$

The function is concave down.

Test for the interval $(0, \frac{4}{3})$ using x = 1:$(-12)(1)(3(1)-4) = 12 \quad (+)$

The function is concave up.

Test for the interval $(\frac{4}{3}, \infty)$ using x = 3: $(-12)(3)(3(3)-4) = -180 \quad (-)$

The function is concave down.

Note that the function does change concavity at the values 0 and $\frac{4}{3}$.

i. $(0, \frac{4}{3})$

j. $(-\infty, 0)$ and $(\frac{4}{3}, \infty)$, because these intervals are not side by side to be considered continous over one large combined interval and in each of these intervals the function is concave up (period).

k. The points of inflection are $(0, 5)$ and $(\frac{4}{3}, \frac{391}{27})$.

Note to obtain the corresponding values for y, you must plug in the relevant values of x into the original function which represents y.

$f(0) = -3(0)^4 + 8(0)^3 + 5 = 5$

$f(\frac{4}{3}) = -3(\frac{4}{3})^4 + 8(\frac{4}{3})^3 + 5 = \frac{391}{27}$

(16) $\ln(12^x) = \ln(e^{x-9})$

$x\ln(12) = (x-9)\ln e$

$x\ln(12) = x - 9$

$x\ln(12) - x = -9$

$x(\ln(12) - 1) = -9$

$x = \frac{-9}{\ln(12)-1} = -\frac{9}{\ln 12 - 1} = -6.0610$

(17) -(19) Refer to tests, handouts, quizzes, and text and Coursecompass.